Quantifying the Residential Demand Response Participation of Dynamic Building Envelopes

Brenton Kreiger *Student Member ASHRAE* **Kyri Baker, PhD** **Wil V. Srubar, PhD**

ABSTRACT

Buildings comprise a large portion of global end-use energy, and within this, usage patterns tend to result in large peaks based on occupancy and weather. These peaks significantly stress the electrical grid via necessary infrastructure upgrades and causing expensive, high emitting generators to ramp up and stay on. This results in the waste, or curtailing, of electricity generated by renewable sources. Load-shifting induced by time-of-use (TOU) electric utility pricing structures, create a demand-response relationship that has an economic benefit for the consumer and the grid as a whole. In this work, a simplified numerical model of a single-family residential heating and cooling system was created to demonstrate the ability of dynamic building envelopes as a part of energy-flexible buildings of the future to participate in this demand-response market. Results illustrate that the best design case can glean 26.3% cost savings above baseline and a 5.9% reduction in monthly peak load using the Boulder, CO time-of-use pricing structure. With hourly optimization, this improved to 35.6% cost savings with the same reduction in monthly peak load. Optimal solar heat gain coefficient (SHGC) and heat transmission (U-value) values vary between 3 main "modes", and this varied allocation illustrates inherently that a dynamic envelope contributes to more optimal building typology settings. With model predictive control, building thermal capacitance becomes available for energy storage, which was not previously considered by hourly optimization. A simplified RC model using MATLAB Simulink showed up to 15.2% yearly cost savings and 4.6% reduction in peak loading moving from a one hour (same as hourly optimization) to eight hour horizon period. A more complex EnergyPlus model developed as a residential protoype building showed, at best, up to 2.5% peak load reduction with a 0.3% increase in yearly costs. Future work will aim to improve the modeling assumptions and explore tuning strategies for this multi-objective optimization problem.

INTRODUCTION

Demand Response Potential

Buildings make up approximately 40% of the entire end-use energy demand in the world on average (Balaras et al. 2007). These numbers will only grow with transitions from oil and gas toward cleaner, electric energy sources, sparking further inquiry into grid resiliency, stability, and building-to-grid interaction. Specifically, incentivizing consumers to change electricity use through demand response programs is one of the most viable methods for reducing peak loads and providing frequency regulation, overall contributing to a more stable and resilient electricity grid. Depending on building typology and climate zone, load-shifting alone can reduce peak loads up to 25% (Oldewurtel et al. 2011) and save the consumer 26% on electricity bills (Henze, Felsmann, and Knabe 2004). Furthermore, structural masses employed as a thermal mass for pre-heating and pre-cooling can reduce residential peaks by 68% (Reynders, Nuytten, and Saelens 2013). Specifically, the present work focuses on residential buildings, which make up 37% of U.S. building energy usage (Yin et al. 2016) and 67% in Europe (Balaras et al. 2007). Residential buildings are generally a target for experimentation with innovative building features and have envelope-dominated building loads, hence, the goal of this study is to quantify the ability of dynamic building envelopes to participate in demand response.

Proposed Work

In contrast to the current state-of-the-art in demand-responsive buildings, where electrical loads are directly controlled to provide demand response services (e.g., cycling air conditioning units during peak grid loading hours), this

Brenton Kreiger is a student in the Department of Civil, Environmental, and Architectural Engineering, University of Colorado, Boulder. **Kyri Baker** is a Assistant Professor in the Department of Civil, Environmental, and Architectural Engineering, University of Colorado, Boulder. **Wil V. Srubar III** is an Assistant Professor in the Department of Civil, Environmental, and Architectural Engineering, University of Colorado, Boulder.

work explores shifting electrical needs by changing the thermal properties of the building during key times. Direct control of electrical loads typically needs battery storage to be cost effective (Oldewurtel et al. 2011), and the resultant load cycling significantly reduces equipment efficiency over time (Waddicor et al. 2016) The authors hypothesized that, with a variety of technologies that allow building envelopes to change setpoint, heat transmission value, and solar heat gain, buildings can become more flexible and significantly impact the demand response (DR) market. Combined with model predictive control (MPC) to encourage pre-heating and pre-cooling effects motivated by a time-of-use rate structure (Garifi et al. 2018), these strategies can potentially save money on the consumer side while contributing to a more resilient and stable electricity grid via peak load reduction. Section 2 first discusses the building typology and cases explored for simulation. Section 3 explains the modeling methodology, as well as objectives and constraints of the model problem. Section 4 discusses the results and impacts from each case. Finally, Section 5 summarizes key conclusions and discusses future work.

MATERIALS

Static Features

The simplified model explores a 2500 square foot (232.3 m^2) home with 10 foot high (3.05 m) walls, generally oriented due south with a square footprint and low-slope roof. The glass heat transmission is considered to have a constant R-value system international (RSI) of 3 (typical for a modern double-paned window with a low emissivity coating), and there is a consistent 30% window-to-wall ratio across all facades with a total standard leakage rate of 0.6 ACH at 50 Pa (0.38 torr) from Passive House Standards (Sadineni, Madala, and Boehm 2011). The location of the residence is considered to be Boulder, CO, with time-of-use pricing adopted directly from the utility, Xcel Energy (Xcel Energy 2019). This comes with a "summer" and "winter" case, where days 91 through 273 of each year are considered summer cases and other days the winter case. In the simplified case, the heating and cooling setpoint is 23 °C (73.4 °F). Yet, in the more advanced analysis, the heating setpoint is 19 °C (66.2 °F) and the cooling set point is 24 °C (75.2 °F) to create a generous deadband that minimally penalizes pre-heating and pre-cooling.

Dynamic Features

This study explores the potential of dynamic envelope features, focusing on material properties, and will be used to assess the *in-situ* ability of dynamic building envelope materials to reduce equipment stress. The main mechanism concerns thermal gains and losses whilst taking advantage of thermal energy storage. Thermal energy storage in residential buildings primarily occurs in the structural mass of the building envelope, floor, and roof, using the "building as a battery" effect. Recently, phase change materials (PCM) and dynamic insulation materials (DIM) have revolutionizied building envelope technologies with their ability to vary heat transmission values and optimize building envelope performance. This innovation has been heavily explored from the load reduction perspective where DIMs can reduce cooling loads 15% on average and 39% at peak potential (Park, Iii, and Krarti 2015) and PCMs use latent heat storage for thermal inertia (Farid et al. 2004). In the simplified model simulated by this study, the thermal resistance value (R-value) of the building envelope is considered to have the potential to change between R-30 and R-40 to represent the ability of operable DIMs or PCMs to dynamically alter the heat transmission values of the building envelope. Additionally, passive solar heating drives the sustainable housing industry (Sadineni, Madala, and Boehm 2011), and new, automated technologies in the building industry, in the form of louvers and electrochromic windows, have the ability to alter lighting from an inelastic to elastic loading in the context of building-to-grid interactions. Electrochromic windows have been show to save 6 to 30 kWh per square foot (Deforest et al. 2017) and adjustable louvers 34% of energy use (Hammad and Abu-hijleh 2010). In the simplified model used in this study, daylighting-based energy savings are not considered, while solar heat gain is considered. In order to represent dynamic solar heat gain, the solar heat gain coefficient (SHGC) is varied reasonably from 25% to 75%. The formulation of solar gains, as well as other features, in the model is discussed further in the methods of Section 3.

METHODS

Input Files and Assumptions

The weather input file for the simulations is an hourly TMY3 weather file from from the *EnergyPlus* database for Boulder, CO. For all cases, thermal resistance is initialized at R-30 and SHGC at 75% with a 23 °C setpoint. Global

horizontal radiation values are processed into direct normal solar radiation on each plane using a sky clearness index algorithm from Skartveit & Olseth (Skartveit and Olseth 1987) to split the horizontal data into direct and diffuse parts, while the Liu & Jordan model for defining the direct normal solar radiation on each plane. These data are modified again based on the window-to-wall ratio (30%), where direct radiation to internal surfaces through windows is considered and sol-air temperature (equations 1 and 2) is used to represent solar heat gain on opaque surfaces.

$$T_{sa} = T_o + \frac{\alpha * \dot{Q}_r}{h} \quad (1)$$

$$h = 10.45 - v + 10v^{0.5} \quad (2)$$

Qr is the solar radiation input, alpha is the surface absorbance taken as a set 0.4, and h is the convective heat transfer coefficient which depends on wind speed, v [m/s](ft/s). Varied model complexity was used throughout the study, increasing in computational intensity and resolution. Each formulation is discussed further in depth, but general assumptions made across all models include:

1. 1D heat transfer through multi-layer opaque surfaces considering only external walls for dynamic analysis.
2. No occupancy and plug loads are considered and immediate switching of dynamic properties occurs.
3. The dynamic range of envelope changes is a reasonable estimate from literature values of the current or near future potential of dynamic building envelopes.

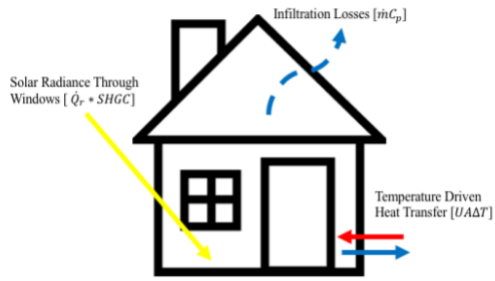

Figure 1 Energy balance across base case model.

Base Case Framework (Simple Model)

The first step in the study was to run a basic analysis of each case varying only SHGC and thermal resistance (R-value). The basic energy balance is depicted in Figure 1, and this model is highly simplified to represent relative changes, where future models (sections below) aim to capture more physical implications of these dynamic changes (*i.e.,* realistic cost and energy usage values). Case comparison was done by looping through each hour of each day of an entire year to calculate hourly costs, which were then summed across the TOU pricing scheme to get yearly cost data and maximized to find peak loads. In each case, the chosen envelope materials remain static and unchanging throughout the year representing "design conditions". These 9 cases are used as a baseline to determine demand response potential through design. The optimal case minimizes the formulation from the Figure 1 energy balance at each hour with the capability of varying U-value and SHGC.

Modeling Framework (Advanced Model)

Control Model Framework. The control model is a physics-based building model that builds off the "Base Case" formulation while preserving quick computation time used for look-ahead purposes. This model considers an efficiency difference in heating and cooling, as well as a lumped capacitance model for temperature degradation over time, using the building time constant (equation 5) determined from building typology.

$$E_{use} = \left(\left(((U * A_{ew}) + (\dot{m} * C_p)) * (T_{set} - T_o)\right) - (SHGC * Q_{sol})\right) * \left(\frac{1}{\eta}\right) \quad (3)$$

$$Cost(t) = E_{use} * \$/kWh \quad (4)$$

Where U is heat transmission in W/m2K (BTU/0Fft2hr), A is area in m2 (ft2), $\dot{m}$ is air mass flow in kg/m3 (lb/ft3), C_p is the specific heat of air, T_s is set point temperature in °C (°F), T_o is outside temperature in °C (°F), SHGC is % solar heat gain, and Q_{sol} is exterior solar gain in W. The lumped capacitance model (equation 6) extrapolates current temperature into a horizon of temperatures if no conditioning were to take place, which allows for pre-cooling and pre-heating benefits to be exploited by summing the cost over the horizon period.

$$\tau = \frac{V * \rho * C_p}{((U * A_{ew}) + (\dot{m} * C_p))} \quad (5)$$

$$T(t) = T_o + \left(\frac{Q_g}{((U * A_{ew}) + (\dot{m} * C_p))}\right) + \left[T_i - T_o - \left(\frac{Q_g}{((U * A_{ew}) + (\dot{m} * C_p))}\right)\right] * e^{-\frac{t}{\tau}} \quad (6)$$

Where V is the house volume in m3 (ft3), ρ is air density at specific elevation in kg/m3 (lb/ft3), T_i is internal temperature in °C (°F) set to the previous timestep interior temperature, Q_g is the heat transfer to setpoint from

equation 3, and t is the time step in hours. The general formulation is a system of linear equations that can be solved quickly and definitively as a *convex* optimization problem.

3.3.2 Optimization Framework. The fundamental optimization problem for all of the models is described in the formulation below. The *sum of the cost vector and maximum of the energy usage vector* are minimized using the built-in MATLAB *fmincon* function, subject to constrained dynamic building envelope variables, where $x(1)$ is the heat transmission of the building envelope (U-value), $x(2)$ is SHGC, and $x(3)$ is the temperature setpoint.

$$\boldsymbol{E}(t) = \left(\left(((x(1) * A_{ew}) + (\dot{m} * C_p)) * (T_{set} - T_o)\right) - \left(x(2) * Q_{sol}\right)\right) * \left(\frac{1}{\eta}\right) \qquad (7)$$

$$\boldsymbol{C}(t) = \boldsymbol{E}(t) * \$/kWh \qquad (8)$$

Minimize: $sum(\boldsymbol{C}^2) + h * \max(\boldsymbol{E})$

Subject To: $\frac{1}{30} < x(1) < \frac{1}{40}, 0.25 < x(2) < 0.75, and\ 19 < x(3) < 24$ (9)

This optimization problem represents a focus on both cost savings (via sum of the cost vector **C**) and reduction in peak demand (via the energy usage vector **E** multiplied by horizon length h in hours) with a slight weight toward peak demand reduction for large horizon periods. However, the weights on these objectives can be tuned depending on individual preferences. The objective function is not changed throughout the study, although a more thorough multi-objective optimization analysis is dicussed in the future work.

Model Predictive Control Logic. The MPC portion of the study evolves from previous results in Section 3.2 in order to further quantify the peak reduction potential and cost savings of dynamic building envelope technologies while considering pre-heating and pre-cooling effects in tandem with dynamic thermal resistance and solar gains. The combination allows for increasing gains, flushing of the space, closing off the space to thermal gains, and more, creating an interesting optimization problem. In the MPC logic, time-of-use pricing is considered with the lumped capacitance temperature degradation equations to incentivize the model for pre-cooling and pre-heating while three optimization variables – thermal resistance, SHGC, and setpoint - are considered. Generally, the control model serves as a quick look-ahead, solved at the next horizon of timesteps, and the optimal results are written to the system model at that timestep to add more physical meaning. Table 1 below summarize the results for the initial testing in which the control model was run as both the system and control model to ensure the goal is met before implementation with longer run times. In Figure 3, the horizon periods of one (solid line), four (dashed), and eight (dotted) hours show increasing lookahead potential where pre-heating occurs and the peak is reduced. This data confirms the MPC model is successful in further reducing peaks and saving money for the consumer at a simplified scale, but a more robust analysis must be performed to add physical meaning to the relative data from above. This is performed both using MATLAB Simulink and EnergyPlus system models.

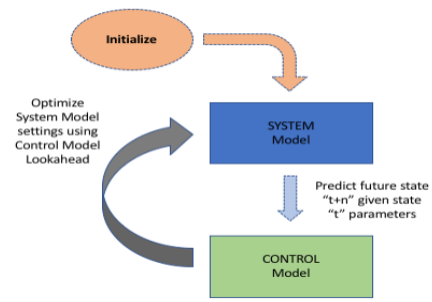

Figure 2 Model predictive control logic.

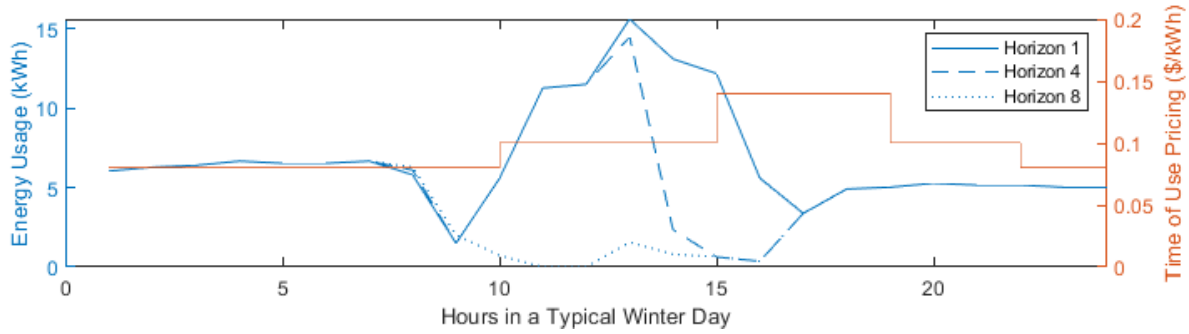

Figure 3 Peak minimization optimization over each horizon period on a typical winter day.

3.3.3 Model 1: Simulink System Model Framework. Now that a working control model is established for look-ahead purposes, a more robust system model is implemented to represent a more realistic physical system. First, a MATLAB Simulink model is considered because this system is easily interfaced with the MPC software written in

MATLAB. This model builds off the base case energy balance and considers (1) a more advanced infiltration model proposed by the National Institute of Standards and Technology (NIST) (Ng, Persily, and Emmerich 2014) based on wind speed local to the site, (2) room element subsystem that uses state space equations to represent the four exterior walls of the building and a simple window model that does not consider incidence angle, (3) room air energy balance of sensible and latent heat using surface temperatures of the interior side of exterior walls and exterior humidity ratio data from the weather file, (4) sol-air temperature to adjust conduction through walls, and (5) a secondary plant model for a convector-radiator heating and chilled ceiling cooling system. The plant model is operated by a PID controller tuned using the Ziegler-Nichols tuning rules for a closed loop system (Ziegler and Nichols 1942) designed for a 25% overshoot response. The main heating system (convector-radiator) is modeled with a stirred tank assumption and split into three zones (third order approach) of equal size with an energy balance at each zone. The emissions index, n, is taken to be 1.3 and the emission constant, K, is determined from the design heating load to the space using a convector radiator specification sheet. The chilled ceiling system is modeled similarly using an emission index of 1 and an online specification sheet assuming 80% of the ceiling is available. This, however, does not completely meet cooling loads for the space, so a simple DOAS system is modeled. When the DOAS system is on, infiltration is a constant rate from ASHRAE 62.1 for 4 people in a space. Also included as subsystems in the plant model are a control valve, actuator, and thermostat. The control valve is modeled as a linear valve with inherent characteristics and installed characteristics considered. Valve let-by is taken as 1% and valve authority is taken as 0.6. The valve actuator is assumed to move at a constant slew rate and hysteresis (or mechanical slack) is considered with a Simulink backslash block. Finally, thermostat delay is considered with first order lag equation assuming a detector time constant of 30 seconds. Generally, the MATLAB Simulink model takes the optimization variables determined using the control model look-ahead each timestep as an input and runs only that current hour in discrete, 3-minute time-steps to determine a more realistic plant energy usage for the hour.

Model 2: EnergyPlus System Model Framework. The EnergyPlus System Model uses a DOE benchmark residential building model developed at the Pacific Northwest National Laboratory (PNNL) for a Colorado Springs based design (Mendon and Taylor 2014). The residence is single family and single-zone over a slab, with a heat pump system and general energy performance conforming to the 2012 International Energy Conservation Code (IECC). The interfacing is done in Python with the "eppy.py" package developed to edit and run EnergyPlus input files (MIT 2004). The MPC script in MATLAB is initiated within a Python script, and the entire protocol is looped within one main Python script. Similarly, the MATLAB control model from above determines optimization variables which are updated in the ".idf" file with eppy and the then run for that timestep; the entire process is then stepped forward one hour and continued throughout the course a year. Inputs and outputs are transferred through Python, and the final data is written to a CSV file for post-processing. Inputs to eppy are the three optimization variables, and outputs are internal temperature in the zone and "heat transfer to set point" for that hour.

RESULTS

Base Case with Optimization

Table 1. Summary of Basic Case Yearly Cost Savings and Peak Reduction

Case [RSI, SHGC]	% Reduction from Max	Peak Reduction (All Hours)	Peak Reduction (3:00 – 6:00 PM)
3 [30, 75%]	**0% (baseline)**	**0% (baseline)**	**0% (baseline)**
7 [40, 25%]	26.25% (best design)	5.9% (best design)	14.8% (best design)
Optimization	35.64%	5.9%	14.8%

Time per run for the "Base Case" is negligible for the static design cases and approximately 90 seconds for a full year of hourly optimization data. The case where SHGC and U-value are optimized hourly provides the highest cost savings from baseline over the course of one year, past the "best design". In this case, the hourly optimization does not make any significant peak reduction over "best design" despite seeing more peak reductions during peak loading times. Figure 4 below represents the fluctuations in cost throughout the year, where the "optimal" case is consistently

lower than all other "best design" cases – shown by the lowest yellow line in the figure. This result is anticipated, given that look-ahead of one hour would just change the building settings to the "best design" case for that hour within given constraints. Possibly increasing the constrained range for the optimization variables would results in further peak reduction from the optimal case. To do this without increasing constraints past what is physicially possible, MPC is used to predict peak loading and make pre-heating and pre-cooling adjustments.

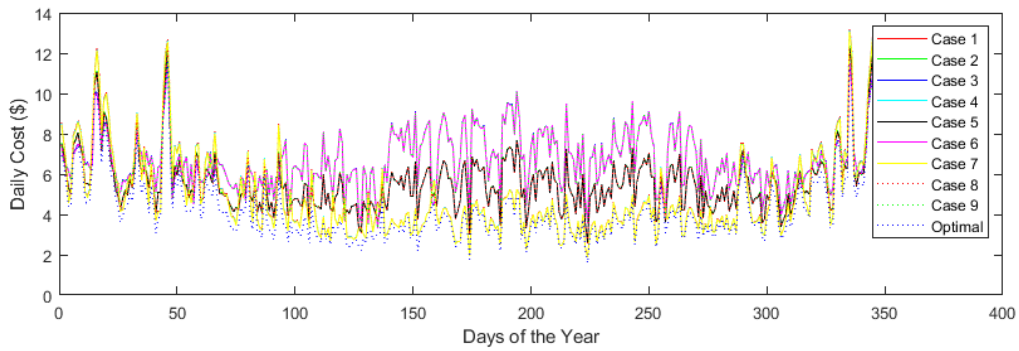

Figure 4 Daily cost data over one year for all design cases plus hourly optimization.

When the optimal case (hourly-based) is considered further, we find some interesting trends in the data by examining the "counts" for each value, or the number of hours that specific building typology is optimal. A wide spread of values confirms the work conceptually, showing that a dynamic building is advantageous. The three "modes" that develop from the optimal model are most likely due to various seasons (*e.g.,* summer, winter, and spring/fall). Figure 5, part c, illustrates that, although the absolute energy peak is not reduced further than the "best design" case with optimization, it remains lower for longer by combining typologies, which makes sense given the cost savings data.

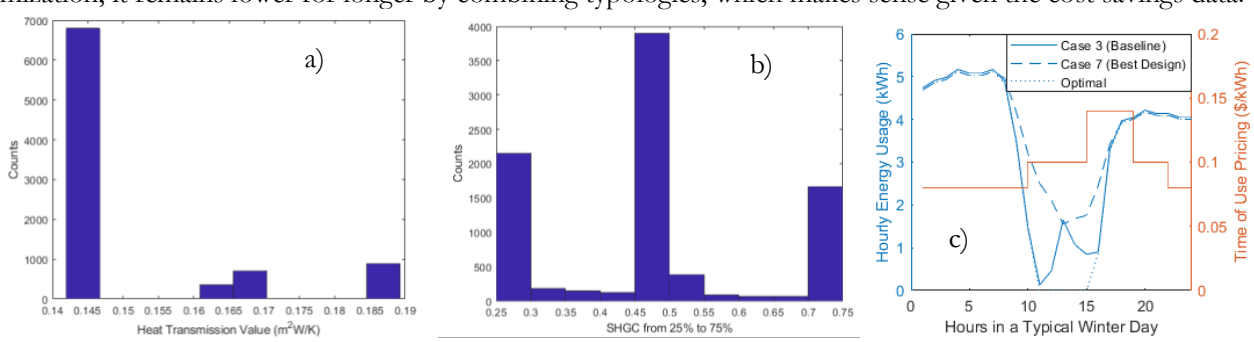

Figure 5 a) Optimal U-values counts. b) Optimal SHGC counts. c) Energy usage versus TOU pricing.

Robust System Model Test Cases

Table 2. Summary of MPC Savings Across Various System Models

Horizon Period	Yearly Cost Savings	Peak Minimization
	Control Model as System Model	
H = 1	**0%**	**0%**
H = 4	26.6%	15.6%
H = 8	49.2%	25.7%
	Model 1: MATLAB Simulink Model as System Model	
H = 1	**0%**	**0%**
H = 4	12.5%	-3.7%
H = 8	15.2%	4.6%
	Model 2: PNNL Residential EnergyPlus Prototype as System Model	
H = 1	**0%**	**0%**
H = 4	-0.3%	2.5%
H = 8	-0.9%	2.5%

Time per run for the control model as the system model is about 150 seconds, where the Simulink model is approximately 35-40 minutes for a full year of data and the PNNL EnergyPlus prototype building takes over 36 hours. The MPC logic utilized in this portion is the same as developed in section 3.3 with various system models that use the optimal building typology determined by the MPC formulation to add more realistic, physical meaning to the otherwise relative savings. Overall, MPC was found to successfully drop peak loads and glean cost savings. The most robust model utilized was the PNNL prototype bulding, a single-family residence for the Colorado Springs, CO area, built to IECC 2012 energy codes.

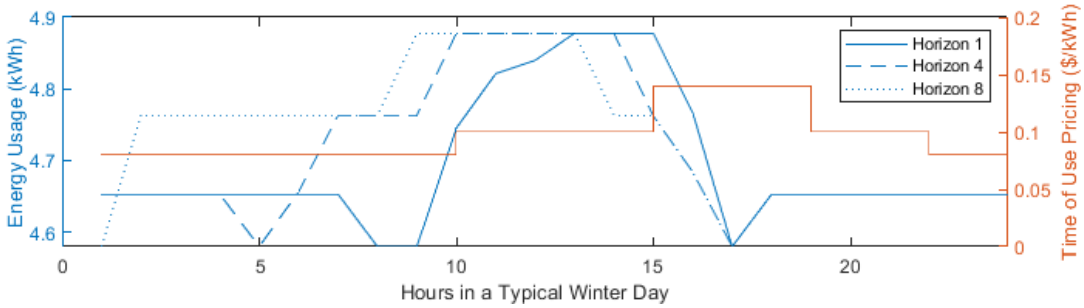

Figure 6 Hourly energy usage showing MPC lookahead the PNNL Model.

In Figure 6, the solid line displays horizon time of one hour, dashed line shows horizon four, and dotted line horizon eight, which illustrates the increasing "look-ahead" ability of the model to reduce loading before TOU pricing (shown in orange as a step function) rises. Overall cost savings in this case decrease slightly, but with benefits to the grid via peak minimization.

CONCLUSION

Summary of Results

First, a case-by-case study was done in conjunction with hourly optimization. The worst case was chosen as the baseline and the best case showed significant cost savings and peak reduction over that baseline discussed above. This can be regarded as savings by "good design". The hourly optimization function, representing the ability of a *dynamic* building envelope to participate in the electricity market, saved even more cost annually – up to 35% - with the same reduction of peak loading (up to 14.8%) as the "best design". This portion of the study shows that hourly optimization, deemed representative of the *ability* of a dynamic building envelope to participate in demand response markets, can push savings past the best, static designs. The next portion of this study utilized a MPC logic evaluated at one, four, and eight hour prediction horizons, in which the summation of costs and maximum energy over the horizon was minimized and a lumped capacity, transient thermal term was used to represent thermal mass effects. Initial results using the simplified, linear control model as the system model show that MPC strategies in conjunction with dynamic building envelopes can further increase cost savings and peak reduction past the best design and optimal cases. This was shown to be 49.2% cost saving and 25.7% peak load reduction over the one hour horizon "hourly optimization" case. This was implemented with both a MATLAB Simulink system model and PNNL EnergyPlus residential prototype building to confirm that cost and/or peak savings are increased with an MPC logic and dynamic building envelope. The Simulink system model showed up to 15.2% cost savings with 4.6% peak reduction, and the EnergyPlus model showed up to 2.5% peak reduction at an annual cost increaseof 0.9%.

Future Work

Improving the Model. First and foremost, validating the model and/or quantifying errors with an experimental component is necessary. A benchmark case for a single-family residence is used, however, due to the lack of data on the subject, the physical implications of dynamic building envelope contribution to grid interaction must be validated experimentally. To improve modeling, a more robust equation that accounts for 2D and 3D heat transfer along with multiple thermal zones (possibly coupling the program with another solver for heat transfer) could potentially glean more accurate results. The authors have also considered a more thorough multi-objective optimization due to the

infrequency of results depending on objective function. Weighting both cost savings and peak reduction, tuned to grid-interactive information could improve the model. Another robust model option for exploration involves using the high-level control mechanisms of the EnergyPlus application "EMS". This allows the user to modify control actions based on sensor values and could potentially be manipulated to simulated a dynamic building envelope. A final improvement on the model suggested for future studies will be to add uncertainty to weather disurbances.

Additional Applications. Dynamic building envelope materials may be more viable to participate in DR when utilized in a larger context. This is worth exploring in depth with an improved model that aggregates multiple residences (*i.e.,* neighborhood) or looks at a single, larger complex (*i.e.,* multi-family building). Beyond peak reduction and cost savings, future work can include frequency regulation. A feasibility study can be done to see if building load-shifting via dynamic envelope technologies can match grid frequency fluctuations in time to provide load disaggregation benefits for the consumer and beneficial grid support to the utility. Also, using this process in the reverse, the future of building envelope materials can be explored by reverse engineering and asking the question, *what building materials can achieve "x" amount of peak reduction, cost savings, and frequency regulation?* Studies can use this foundational concept to define and optimize material variability such that demand response participation improves grid resilience and has a marketable ROI. Finally, considering hygroscopic materials is another important layer. Moisture content affects essential material properties for energy modeling (*i.e.,* thermal conductivity, thermal resistance, density), and is usually viewed as a detrimental model mismatch. Taking advantage of hygroscopic properties with MPC can potentially provide advantageous energy flexibility benefits in the future through thermal energy storage.

ACKNOWLEDGMENTS

This research was made possible by the Department of Civil, Environmental, and Architectural Engineering, at the College of Engineering and Applied Sciences at the University of Colorado Boulder. This work represents the views of the authors and not necessarily those of the sponsors.

NOMENCLATURE

T_{sa} = Sol-air temperature [°C](°F)

T_o = Outdoor air temperature [°C] (°F)

a = Absorptivity [%]

h = Convective heat transfer coefficient [W/m2K](BTU/°F ft2hr)

$\dot{Q}_r$ = Solar radiation [W/m2](BTU/ft2hr)

v = Wind speed [m/s](ft/s)

U = Conductive heat transmission [W/m2K] (BTU/0Fft2hr)

A_{ew} = External wall area [m2](ft2)

$\dot{m}$ = Infiltration air mass flow rate [kg/m3](lb/ft3)

C_p = Specific heat of air [kJ/kgK](BTU/lb°F)

T_{set} = Set-point temperature [°C] (°F)

$SHGC$ = Solar heat gain coefficient [%]

Q_{sol} = Solar radiation on a vertical plane [W/m2] (BTU/ft2hr)

η = efficiency [%]

$C(t)$ = Cost at time t [$]

E_{use} = Energy use [kWh](BTU)

A = Valve authority [%]

V = Volume of the room [m3](ft3)

ρ = Density [kg/m3](lb/ft3)

$T(t)$ = Temperature at time t [°C] (°F)

T_i = Internal temperature [°C] (°F)

τ = Building time constant [1/h]

Q_g = Heating gains for design load purposes [W](BTU/hr)

C_w = Combined heat capacity of equipment and water [W/m2K] (BTU/°F ft2hr)

T_{w1}, T_{w2}, T_{wr} = Stirred tank temperatures [°C] (°F)

m_w = Mass flow rate of water through heating or cooling equipment [kg/m3](lb/ft3)

K = Emission constant for heating or cooling equipment [W/m2K] (BTU/°F ft2hr)

n = Emission index for heating or cooling equipment [Unitless]

G = Inherent valve characteristics [Unitless]

G' = Installed valve characteristic [Unitless]

f_o = Valve let-by [%]

p = Valve position [%]

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